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THE BEGINNING OF FORMAL NUMBER WORK

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Most courses of study make work in number incidental in the first year. By this provision the teaching of number depends at first on the needs which the school presents for the use of number. These are really very few. Counting the number of pupils in a row for the distribution of materials is a genuine demand for the use of number. The "helper," having six pupils in his row, brings six boxes and gives a box to each pupil. Counting the number of words as the vocabulary increases and the number of pupils present each half-day is a means of determining facts of interest. This counting may serve as incentive for adding to vocabulary and for regularity in attendance. The number work for the first half-year is usually limited to oral counting of objects to meet number demands of the school.

In the second half of the first school year the oral counting to meet school needs is continued, and ingenuity is exercised to use number, without taxing the pupil's ability, in a way to lay a foundation for subsequent work. The pupils learn to give the number of the pages of their reading-books. They learn to recognize the printed names of numbers from one to ten and the figures as representing numbers. They learn to make figures and by the use of inch sticks, lentils, or pictures indicate concretely the number for which each stands. This is merely perceptual learning. The process in learning to recognize the word "four" or the figure "4" is not different from that in learning to recognize the word "bird."

The second school year usually begins dealing more formally with number relations and attempts abstraction. In the second (and each subsequent year in the primary grades) the teacher must not lose sight of the two phases of number work—that of using number for immediate needs and that of gaining control over the symbols of number. Advantage should be taken of all opportunities for using number. They occur in measuring and making booklets, seed envelopes, card boxes, calendars, in other forms of construction, and in counting and keeping the score in games. The formal work may be introduced in answer to some need. A pupil must learn to make figures well and rapidly, but the immediate end may be set up of learning to make figures well and rapidly in order to keep the score in games. The formal teaching of number begins, practically, in the second year. The pupil's equipment is the ability to count objects and to recognize figures. This ability varies in different pupils, but in each case it forms the basis from which to proceed. Counting, repeating the number names in a series, is the primitive response to a "how many" stimulus. It is customary to teach second-grade pupils to count to one hundred orally and to write figures to one hundred. In the writing, for convenience, the figures are often grouped in tens.

It is possible to use this device much more effectively than is usually done. If, before emphasizing the oral counting, the pupil can have in his hand a number scale in large, clear figures, as in Table I,[1] time will be economized.

The counting should be first from this scale, so that each number name corresponds to a symbol. When the child says "eleven, twelve, thirteen, fourteen," he looks at the figures "11, 12, 13, 14." He thus organizes the numbers 1–100, so that the numbers have certain space relationships in his mind and can be easily recalled. The work on *form* must follow and grow from work which is concrete. These tables, through the visualization of symbols and space relationships of numbers, are designed to assist in the process of abstraction. If in counting from memory the pupil hesitates, he should look at once at his number scale, and, as in the beginning, read the figures, after which he can repeat them readily. Visualization should be emphasized in the early formal work.

The first counting should be by 1's; begin at 1, count to 100. Follow by counting in groups of 10; in the last column, J, begin with 10, count to 100; first column, A, begin with 1 and count by

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[1] The exact style of figures to be used is shown in Table I. In the other tables a different type is used to save space.

10's to 91; in the same way count each column by 10's. Count each column in reverse order, for example, in column B begin at 92, count to 2. Pupils should learn to write figures to 100. An immediate use is the making of inch-square number cards on gray bogus paper. These are kept in boxes or trays and furnish a profitable form of seat work. The pupils place the cards on the desk to show the groups of 10's, A–J.

TABLE I

NUMBER SCALE

	A	B	C	D	E	F	G	H	I	J
1)	1	2	3	4	5	6	7	8	9	10
2)	11	12	13	14	15	16	17	18	19	20
3)	21	22	23	24	25	26	27	28	29	30
4)	31	32	33	34	35	36	37	38	39	40
5)	41	42	43	44	45	46	47	48	49	50
6)	51	52	53	54	55	56	57	58	59	60
7)	61	62	63	64	65	66	67	68	69	70
8)	71	72	73	74	75	76	77	78	79	80
9)	81	82	83	84	85	86	87	88	89	90
10)	91	92	93	94	95	96	97	98	99	100

For diversity in grouping Table II is given. Each group of 10's is arranged in vertical instead of horizontal lines. It is used in the same way.

Following the counting by 1's, 10's, and 5's, count by 2's. Beginning with zero touch, while counting, the even numbers; beginning with 1 touch, while counting, the odd numbers. Subsequently count from memory. Count by 3's and by 4's through the first three or four groups. Count by adding to lines A–J numbers 1 to 9, as, add 4 to each number in line E: 5 plus 4 equal 9, 15 plus 4 equal 19, 25 plus 4 equal 29, to 95 plus 4 equal 99; subtract, similarly, as in Table I, column C, 3 minus 3 equals zero, 13 minus 3 equals 10, to 93 minus 3 equals 90. Following actual subtraction with objects this scale may be used to advantage with pupils who are unable to make the abstraction readily. For example, in the problem 9 minus 5, begin at 9 in row (1); count back 1, 2, 3, 4, 5; the figure 4 indicates the number that remains; also, the places 1, 2, 3, 4 show it.

TABLE II

NUMBER SCALE

A—	1	11	21	31	41	51	61	71	81	91
B—	2	12	22	32	42	52	62	72	82	92
C—	3	13	23	33	43	53	63	73	83	93
D—	4	14	24	34	44	54	64	74	84	94
E—	5	15	25	35	45	55	65	75	85	95
F—	6	16	26	36	46	56	66	76	86	96
G—	7	17	27	37	47	57	67	77	87	97
H—	8	18	28	38	48	58	68	78	88	98
I—	9	19	29	39	49	59	69	79	89	99
J—	10	20	30	40	50	60	70	80	90	100

The intention of Table III is to lessen the difficulties of learning the addition combinations by building them up by counting and by showing them all upon one page. It often seems that pupils become lost in the maze of addition combinations and experience the uncertainty and distress of one who has lost his way. The pupil does not know what new combinations may leap out at him from the region of unknown things. But if he sees the primary sums, *all* of them, and learns that there are only forty-five, the task of learning them becomes much more definite and possible. After the combinations have been learned by counting, as in columns A to I, Table III, is useful in speed drills. A class goes to the board with instructions to write column (3). The pupils begin at a signal. The one who finishes first says "1," and makes the figure "1" over his work. Each, as he finishes, announces and records his place.

The teacher checks on accuracy. At the end of the recitation it is of interest to observe the different scores. The teacher should try to account, to herself at least, and by observation while the work is in progress, for the variation in speed of a given pupil, for example, if the score is 1–4–2–1, why did the pupil drop to fourth place in the second exercise? In using Table III at the desks the pupils build the primary sums with their inch-square number cards, varying the order each day.

TABLE III

THE 45 PRIMARY SUMS

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
A	1 1	2 1	3 1	4 1	5 1	6 1	7 1	8 1	9 1
B	2 2	3 2	4 2	5 2	6 2	7 2	8 2	9 2	
C	3 3	4 3	5 3	6 3	7 3	8 3	9 3		
D	4 4	5 4	6 4	7 4	8 4	9 4			
E	5 5	6 5	7 5	8 5	9 5				
F	6 6	7 6	8 6	9 6					
G	7 7	8 7	9 7						
H	8 8	9 8							
I	9 9								

Subtraction is developed by counting, using Table III, as 1 minus 1 equals zero, 2 minus 1 equals 1, continuing through columns A to I.

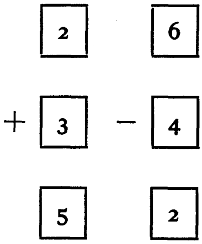
Table IV presents the differences in varying order. The pupils are encouraged to arrange the "problems" in a different order each day when placing them on their desks with inch-square number cards. It is very important that the addition and subtraction processes be closely associated. When 4 and 3 are combined to make 7, at once separate each addend from the sum; 7 minus 3 equals 4, 7 minus 4 equals 3.

TABLE IV

THE 45 PRIMARY DIFFERENCES

	1	2	3	4	5	6	7	8	9
A	1	2	3	1	2	4	1	3	1
B		1	2	4	5	2	5	2	7
C			1	3	3	5	3	1	9
D				2	4	3	4	6	4
E					1	6	2	8	5
F						1	7	4	6
G							6	7	3
H								5	8
I									2

After inspection and correction of "problems" laid upon the desk with inch-square number cards, the inch sticks being used to indicate (+, −) the sums and differences, an exercise to increase speed may be given. The pupils are instructed to gather the answer cards into a heap, thus mixing them, leaving the problems in orderly rows on the desks. At a signal the pupils begin placing the "answer" cards in position. Each pupil stands as soon as he has accomplished this task. The teacher keeps the score. On a properly ruled chart upon which the pupils' names are written, she records the order in which the pupils stand and the time in minutes or seconds. In trying to be first to stand, each pupil gets a review of the whole number lesson and discovers the combinations which retard him most. He has now a motive for drilling upon these. He is anxious to know all the combinations before



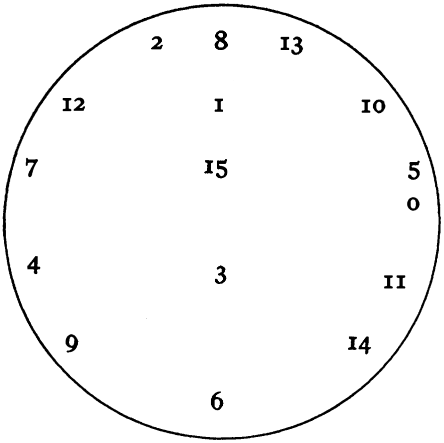
"Problems"

another trial. In subsequent speed exercises pupils who do not work rapidly enough to stand among the first still have the satisfaction of improving their record by decreasing the time required to do the work.

Following the counting for becoming familiar with the number spaces 1–100 should come counting by 2's, 3's, 4's, etc., within the first two groups of ten, as an aid in learning the combinations. For example, the pupil counts by 4:

beginning with zero:	0, 4, 8, 12	in reverse order:	12, 8, 4, 0
1:	1, 5, 9, 13		13, 9, 5, 1
2:	2, 6, 10, 14		14, 10, 6, 2
3:	3, 7, 11, 15		15, 11, 7, 3

At first the pupil uses Tables I and II if necessary. Soon he has the series memorized so that he can speak the numbers rapidly. To test his ability to use the series in adding and subtracting make a drill device as in the circle below.



Use clear, black figures at least $1\frac{1}{2}$ inches high, cut from a wall calendar. Paste them in mixed order upon a piece of bogus paper 12×18 inches. The pupil, with a pointer, adds and subtracts numbers as directed. If the requirement is to add 4 to each number in the circle and the pupil cannot at once speak the sum of 8 and 4, he drops back to the series 0, 4, 8, 12 and gives the sum, "8 plus 4 equal 12." If his difficulty is in subtracting 4 from 11, he repeats the series 3, 7, 11—11, 7, 3, and states, "11 minus 4 equals 7." As rapidly as they become able to do so, pupils are expected to give sums and differences in this drill exercise without naming the numbers which make up the combinations.

Table V gives further drill in addition and subtraction. Sometimes the pupils read the problems, as 4 plus 3 equal 7; at other times they simply name the sums and differences. The purpose of this work is to make pupils able to recognize the primary combinations instantly, and to speak or to write them instantly. The use of perception cards (3×5½ inches on which each figure is at

TABLE V

Addition

Read sign + plus. Read 0, zero.

4	1	5	2	3	4	7	0	2	0	5	7
+3	2	4	7	2	0	1	5	4	8	5	3
4	9	2	6	4	0	3	6	1	6	3	9
+2	1	0	1	5	0	4	3	5	2	6	0
8	6	5	3	9	4	2	4	8	7	6	9
+3	9	2	7	7	1	2	7	5	4	0	9
3	0	6	7	8	9	0	7	10	8	6	8
+9	10	4	5	6	2	11	8	4	9	5	7
9	5	8	9	7	9	6	3	0	7	4	5
+8	6	7	3	7	6	7	5	7	6	7	8

Subtraction

Read sign − minus. Read 0, zero.

6	7	8	4	5	7	9	11	2	12	8	6
−2	5	1	2	0	3	2	5	0	5	3	3
11	8	3	13	14	9	7	12	4	18	10	3
−2	5	0	5	7	4	3	6	1	9	7	2
15	9	7	13	10	8	12	9	1	16	7	4
−7	3	1	7	5	4	5	6	0	8	2	0
12	9	7	17	15	7	11	13	15	14	12	9
−7	5	0	9	8	6	6	8	6	6	2	0
10	11	9	5	17	15	10	14	9	11	8	14
−2	4	1	1	8	9	3	4	7	7	0	9

least an inch and a half high) is valuable. The teacher exposes the card an instant; the pupil gives the sum or difference as directed. If he misses, he takes the card and reports the combination correctly when he hands it back at the close of the

PERCEPTION CARD

6

2

recitation. By shifting the perception cards the teacher makes sure that the pupils are able to recognize the combinations in any order.

TABLE VI

Doubles of Numbers

Read signs: + plus, − minus.
Learn doubles of numbers.

1	2	3	4	5	6	7	8	9	10	11	12
+1	2	3	4	5	6	7	8	9	10	11	12

Use doubles of numbers as a guide in addition and subtraction.
Read from top: as, 2 plus 3 equal 5.

2	2	2	3	3	3	4	4	4	5	5	5
+2	3	1	3	4	2	4	5	3	5	6	4
6	6	6	7	7	7	8	8	8	9	9	9
+6	7	5	7	8	6	8	9	7	9	10	8
10	10	10	11	11	11	12	12	12			
+10	11	9	11	12	10	12	13	11			

Read from top: as, 6 minus 4 equal 2.

2	3	4	4	4	5	6	6	6	7	8	8	8		
−1	−3	−2	3	1	−5	−3	4	2	−7	−4	5	3		
9	10	10	10	11	12	12	12	13	14	14	14			
−9	−5	6	4	−11	− 6	7	5	−13	−7	8	6			
15	16	16	16	17	18	18	18	19						
−15	− 8	9	7	−17	− 9	10	8	−19						
20	20	20	21	22	22	22	23	24	24	24				
−10	11	9	−21	−11	12	10	−23	−12	13	11				

Table III, column (1) presents the doubles of numbers below ten. They are easily learned in the forty-five primary sums of which they constitute one-fifth. The knowledge of these furnishes a guide in addition and subtraction. Table VI suggests the use of this knowledge in learning and recalling primary sums and differences. For instance, 8 plus 8 equal 16; 7 is 1 less than 8; hence 8 plus 7 equal 1 less than 16, or 15; 8 plus 8 equal 16; 9 is 1 more than 8; hence 8 plus 9 equal 1 more than 16, or 17. When the pupil has the idea of using the double of a number as a guide in addition or subtraction he is able to do it instantly without any formal explanation.

Counting by 10's is an easy form of adding or subtracting. Pupils often consider adding 9 difficult. This difficulty may be removed by using 10 as a guide in adding 9 as suggested in Table VII.

TABLE VII

Addition

Ten. It is easy to add ten to any number. Read lower number first: as 1 plus 10 equal 11.

10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
+1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Nine. 9 is 1 less than 10. The sum of 9 and any number is 1 less than the sum of 10 and that number. Use 10 as a guide in adding 9 to any number.

10	9	10	9	10	9	10	9	10	9	10	9	10	9	10	9
+2	2	+3	3	+4	4	+5	5	+6	6	+7	7	+8	8	+9	9

TABLE VIII

Table of Two's

1	2	3	4	5	6	7	8	9	10	11	12
2	2	2	2	2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2	2	2	2	2
	4	2	2	2	2	2	2	2	2	2	2
		6	2	2	2	2	2	2	2	2	2
			8	2	2	2	2	2	2	2	2
				10	2	2	2	2	2	2	2
					12	2	2	2	2	2	2
						14	2	2	2	2	2
							16	2	2	2	2
								18	2	2	2
									20	2	2
										22	2
											24

In the counting by 2's from the number scales, Tables I and II, the equivalent of the multiplication table of two's has been developed. These facts may be thrown into the form of Table VIII. Use this for both multiplication and division: 9 times 2 equal 18. How many 2's in 18? There are nine 2's in 18.

SUMMARY

Formal work in number usually begins in the second year.

At first the work must be concrete.

Strong visualization of symbols and of space relationships of numbers assists in the process of abstraction.

At the end of the second year, pupils should have a clear image of numbers from 1 to 100 in groups of ten, and the relationship of these numbers one to the other; and instant and automatic command of the primary sums and differences.